$\boxed{\text{oef } 12.6 \text{ variant}}$ 12.6.b zijn geen commuterende matrices en dus zal simultaan diagonaliseren niet mogelijk zijn. We zullen volgende variant oplossen:

$$A = \begin{pmatrix} 5 & 1 & -1 \\ 1 & 4 & 0 \\ -1 & 0 & 4 \end{pmatrix} ; B = \begin{pmatrix} 11 & 1 & -1 \\ 1 & 6 & -4 \\ -1 & -4 & 6 \end{pmatrix}$$

eigenwaarden van A:

$$\begin{vmatrix} 5-\lambda & 1 & -1 \\ 1 & 4-\lambda & 0 \\ -1 & 0 & 4-\lambda \end{vmatrix} = -(4-\lambda) + (4-\lambda)\left((5-\lambda)(4-\lambda)-1\right)$$

$$= \lambda - 4 + (4-\lambda)(\lambda^2 - 9\lambda + 19)$$

$$= (4-\lambda)(\lambda-3)(\lambda-6)$$

$\leadsto$ eigenwaarden van $A = \{4, 3, 6\}$

Merk op: ze hebben allemaal multipliciteit 1 en dus ~~hebben~~ zullen hun eigenruimte dimensie 1 hebben. Bijgevolg zullen we nooit 'stap 3' moeten uitvoeren.

We kiezen de eigenwaarde $\lambda = 3$:

$$V_3 = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \;\middle|\; A \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} \overset{(*)}{=} 3 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \right\}$$

(*) oplossen:
$$\left(\begin{array}{ccc|c} 2 & 1 & -1 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\leadsto \begin{cases} x = z \\ y = -z \end{cases}$$

$$\text{Dus } V_3 = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \;\middle|\; \begin{matrix} x = z \\ y = -z \end{matrix} \right\} = \left\{ \begin{pmatrix} z \\ -z \\ z \end{pmatrix} \;\middle|\; z \in \mathbb{R} \right\}$$

$$= \text{vect}\left\{ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\}$$

$$\text{Neem } \vec{b}_1 = \frac{(1,-1,1)}{\|(1,-1,1)\|} = \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

oef 12.6 variant vervolg | Vervolgens zoeken we de 2 andere basis vectoren binnen $V_3^\perp$:

$$V_3^\perp = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \,\middle|\, \left\langle \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \vec{v} \right\rangle = 0, \forall \vec{v} \in V_3 \right\}$$

$$= \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \,\middle|\, \left\langle \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\rangle = 0 \right\}$$

$$= \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \,\middle|\, x - y + z = 0 \right\} = \text{vect}\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$$

De tweede basis vector vinden we binnen:

$$V_3^\perp \cap V_4 = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \,\middle|\, \begin{array}{l} x - y + z = 0 \\ A \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} \underset{(**)}{=} 4 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \end{array} \right\}$$

(**) oplossen:

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\Rightarrow \begin{cases} x = 0 \\ y = z \end{cases}$$

$$\Rightarrow V_4 = \left\{ \begin{pmatrix} 0 \\ y \\ y \end{pmatrix} \,\middle|\, y \in \mathbb{R} \right\} = \text{vect}\left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$$

dus

$$V_3^\perp \cap V_4 = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \,\middle|\, \begin{array}{l} x = 0 \\ y = z \\ x - y + z = 0 \end{array} \right\}$$

$$= \left\{ \begin{pmatrix} 0 \\ y \\ y \end{pmatrix} \,\middle|\, y \in \mathbb{R} \right\} = \text{vect}\left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\} = V_4$$

We nemen nu $\vec{b_2} = \dfrac{(0,1,1)}{\|(0,1,1)\|} = \left(0, \dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$

Tenslotte om $\vec{b_3}$ te vinden werken we binnen:

$$V_3^\perp \cap V_4^\perp \cap V_6$$

oef 12.6. variant vervolg

$$V_6 = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \;\middle|\; A \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 6 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \right\}$$

$$\left(\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 1 & -2 & 0 & 0 \\ -1 & 0 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 0 & -2 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} -1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & \end{array}\right)$$

$$\rightsquigarrow \begin{cases} x = -2z \\ y = -z \end{cases}$$

$$= \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \;\middle|\; \begin{array}{l} x = -2z \\ y = -z \end{array} \right\}$$

Verder $V_3^\perp \cap V_4^\perp = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \;\middle|\; \begin{array}{l} \underbrace{x - y + z = 0}_{\text{komt van } V_3^\perp} \\ \underbrace{\left\langle \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\rangle = 0}_{\text{komt van } V_4^\perp} \end{array} \right\}$

$$= \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \;\middle|\; \begin{array}{l} x - y + z = 0 \\ y + z = 0 \end{array} \right\}$$

$$\Rightarrow V_3^\perp \cap V_4^\perp \cap V_6 = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \;\middle|\; \begin{array}{l} y = -z \\ x = -2z \\ x - y + z = 0 \end{array} \right\}$$

$$= \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \;\middle|\; \begin{array}{l} y = -z \\ x = -2z \end{array} \right\} = \text{vect}\left\{ \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} \right\}$$

Dus we nemen $\vec{b}_3 = \dfrac{(-2,-1,1)}{\|(-2,-1,1)\|} = \left(\dfrac{-2}{\sqrt{6}}, \dfrac{-1}{\sqrt{6}}, \dfrac{1}{\sqrt{6}} \right)$

Alles samen verkrijgen we de matrix M !

$$M = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & -2/\sqrt{6} \\ -1/\sqrt{3} & 1/\sqrt{2} & -1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \end{pmatrix}$$